Structural Change, Growth and Volatility*

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Abstract

I construct a two-sector general equilibrium model of structural change to study the impact of sectoral composition of GDP on cross-country differences in GDP growth and volatility. For an empirically relevant parametrization of sectoral production functions, an increase in the share of services in GDP reduces both aggregate TFP growth and volatility, thus reducing GDP growth and volatility. When the model is calibrated to the US manufacturing and services sectors, the rise of the services sector occurring as income grows can account for a large fraction of the differences in per-capita GDP growth and volatility between high- and upper-middle-income economies.

JEL Classification: C67, C68, E25, E32.

Keywords: Structural Change, Growth, Volatility, Total Factor Productivity.

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1 Introduction

This paper puts forth the idea that the composition of GDP represents an important channel in shaping both GDP growth and volatility. Cross-country evidence suggests that: i) per-capita GDP of high income economies grows slower than that of middle income economies; ii) high income economies display lower per-capita GDP volatility than middle income ones; and iii) the share of services in GDP increases with income per-capita. These facts together suggest that both the growth rate and the volatility of an economy might be related to its productive structure. To address this issue, I first report empirical evidence on the three observations above. Next, I present a two-sector general equilibrium model qualitatively consistent with such evidence. Finally, I use a calibrated version of the model to assess the importance of structural change between the broadly defined manufacturing and services sectors for observed differences in per-capita GDP growth and volatility between upper-middle and high income economies. Even in the most conservative case a substantial portion of the differences in growth and volatility between the two groups of countries can be accounted for by structural change.\(^1\)

I find that the negative relationship between the share of services and per-capita GDP growth emerges after a certain income level. In particular, by sorting countries according to the income classification of the World Bank, I find that this relationship is statistically significant for countries included in the High and Upper-Middle Income groups. Instead, the relationship disappears for Lower Middle and Low income economies. Regarding volatility, there is a negative relationship with the share of services at all income levels, which confirms previous findings that volatility declines monotonically with development.

To account for these empirical findings, a model of structural change in which growth and volatility both depend on the size of the services sector is needed. The model in Moro (2012) displays a negative relationship between the share of services and GDP volatility. Here I use a similar setup and show that structural change has an effect on GDP growth which mimics the one on volatility. A sketch of the model is as follows. Gross output in each of the two sectors is produced by combining labor, manufactured intermediate goods and intermediate services. This environment implies the existence of a well defined two-by-two input-output

\(^1\)In this paper I abstract from explicitly modelling the agricultural sector. The aim here is to compare per-capita GDP growth and volatility of economies that already moved away from being mainly agricultural.
structure. Gross output TFP grows according to a common stochastic process in the two sectors and households have Stone-Geary preferences that imply an income elasticity larger than one for services consumption. As GDP increases because of TFP growth, the model endogenously generates a rise in the share of services in GDP.

In models with input-output linkages, intermediate goods amplify the effect of gross output TFP changes on GDP. To see this, consider the economy described above as governed by a benevolent social planner and assume a 1% TFP increase common to both sectors. By using the same amount of labor and intermediates as before, each sector can now produce a 1% larger amount of gross output. Assume that the planner allocates the gross output of each sector in constant proportions to final demand and to intermediate goods supply. Then, the 1% TFP increase leads to a 1% increase in GDP and to a 1% increase in intermediate goods provision. These additional intermediates allow a further increase in sectors’ gross output and, in turn, in GDP. As a result, the response of GDP to the increase in gross output TFP is amplified through intersectoral linkages and is larger than 1%. In turn, the magnitude of the amplification mechanism depends on the importance of intermediate goods in production. The more intensive is technology in intermediates goods, the larger the elasticity of gross output with respect to intermediate inputs. Thus, the elasticity of GDP to gross output TFP, which determines GDP growth and volatility, is an increasing function of the intensity of intermediate goods in production.

When the two sectors in the economy display a different intensity of intermediates the elasticity of GDP to sectoral TFP becomes an endogenous variable which depends on sectors’ relative size. In particular, GDP growth and volatility increase with the size of the sector with the largest intensity of intermediate goods in production. In this paper I provide novel evidence that the share of intermediate goods in gross output is larger in manufacturing than in services in 26 developing and developed economies over the 1970-2005 period. Thus, for a common sectoral TFP process in the two sectors, when production functions in manufacturing and services are calibrated to data from any of these countries, the model delivers a smaller GDP growth and volatility the larger the size of the services sector. This, in turn, implies that structural change towards services induces an endogenous decline in GDP growth and volatility along the growth path.

To perform the quantitative analysis, I use data from Upper-Middle and High income countries, for which I find a statistically significant relationship between the share of services
and both growth and volatility. The average share of services in Upper-Middle income economies increases from 0.46 to 0.56 during the 1970-2010 period while that of High Income economies increases from 0.54 to 0.69 during the same period. Also, Upper-Middle income economies display an average growth rate of 2.57% per-year while High income one of 2.30%. Volatility, measured as the standard deviation of per-capita GDP growth rates over the period, is 3.82% for Upper-Middle income and 3.06% for High income. To compare model with data, I study an economy calibrated such that it generates a transition path in which the share of services increases from the level displayed by Upper-Middle income economies at the beginning of the sample (0.46) to that displayed by High income economies at the end of the sample (0.69). I consider both an economy with a common gross output TFP process across the two sectors and one in which gross output TFP processes are calibrated to U.S. data from manufacturing and services sectors. The most conservative result suggests that around 40% of the differences in growth and around 50% of the differences in volatility between Upper-Middle and High income economies during the 1970-2010 period can be accounted for by a unique model economy at different stages of development.

In the model I exploit the mechanism linking intermediate goods utilization to aggregate TFP first shown in the theoretical work of Hulten (1978). Recently, several contributions studied the implications of explicitly considering this channel for macroeconomic outcomes: Ciccone (2002), Ngai and Samaniego (2009), Jones (2011), Moro (2012) and Acemoglu et al. (2012) among others. In Moro (2012) I show that in the U.S. the share of intermediate goods is larger in manufacturing than in services in the post-war period. This implies that, when services grow relative to manufacturing, the intermediate goods multiplier associated to aggregate TFP endogenously declines. In the calibrated model this mechanism can explain 28% of the decline in GDP volatility between the 1960-1983 and 1984-2005 periods. Here I use a similar model to show that, together with volatility, the endogenous change in the aggregate TFP multiplier due to structural change also affects the growth rate of the economy. This result, together with novel cross-country evidence showing that the share of intermediates in gross output is larger in manufacturing than in services, allows me to show that a larger share of services implies smaller aggregate GDP growth and volatility in the

\footnote{The decline in output volatility has been extensively studied for the U.S. [McConnell and Perez-Quiros (2000), Blanchard and Simon (2001) and Stock and Watson (2002)] but with little attention to the sectoral composition explanation, with Alcalá and Sancho (2003) and Carvalho and Gabaix (2013) representing important exceptions.}
theoretical model.\footnote{Regarding the transition from agriculture to manufacturing, Echevarria (1997) shows that this process implies an increase in the GDP growth rate, while Da-Rocha and Restuccia (2006) show that a decline in the share of agriculture in GDP implies a reduction in GDP volatility.}

In the literature on cross-country evidence on economic growth, Lucas (1988) notes that growth rates of middle income countries are larger than those of high income ones. Echevarria (1997) provides empirical evidence on this fact, and also shows that the share of services in GDP increases with income.\footnote{This last fact is confirmed in panel regressions that control for countries fixed effects in Eichengreen and Gupta (2011).} On the theory side, several papers study the relationship between output composition and economic growth.\footnote{Baumol (1967), Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) are classical references.} Echevarria (1997) presents a three-sector structural change model and calibrate it to study the effect of the structural composition on the GDP growth rate. She shows that, if value added TFP in manufacturing grows faster than in services, a structural change from manufacturing to services implies a decline in the growth rate of GDP. The theoretical construct presented in this paper is similar to Echevarria (1997), with one important distinction. By explicitly considering intermediate goods in production, I am able to show that structural change towards services implies a reduction in aggregate GDP growth even when sectoral TFP growth is the same in both sectors.\footnote{In the context of the convergence literature, the model in this paper exhibits $\beta$-convergence, that is, given the same sectoral TFP growth in manufacturing and services, poorer countries (that have a smaller share of services) grow faster that richer ones. For a definition of $\beta$-convergence see Sala-i-Martin (1996).} In addition, this mechanism allows me to show that the effect of structural transformation is the same on both GDP growth and volatility, while Echevarria (1997) focuses on GDP growth.

Regarding GDP volatility, Lucas (1988) highlights its negative relationship with the level of income. Acemoglu and Zilibotti (1997) provide robust empirical evidence on this while Koren and Tenreyro (2013) report that GDP volatility declines with development both in a cross-section of countries and for each country over time. These empirical findings are consistent with the qualitative predictions of the model presented here. Also, Koren and Tenreyro (2007) find that the sectoral composition can account for up to 60\% of the difference in aggregate volatility between poor and rich countries. In the most conservative specification I find that roughly one-half of the difference in volatility between High and Upper-Middle income countries can be accounted for by the increase in the services sector relative to manufacturing occurring during the process of development.
The theoretical literature on the decline of GDP volatility along the development path focuses mainly on financial and technological diversification arguments. Acemoglu and Zilibotti (1997) present a model in which volatility is high at early stages of development because of a lack of diversification in investment projects. Koren and Tenreyro (2013) propose a theory based on technological diversification to explain the decline in both aggregate and firm level volatility during the development process.\footnote{See also Jaimovich (2011) for a theory of development with similar predictions.} Though both works focus on the role of intermediate inputs in shaping aggregate volatility, the mechanism proposed in this paper is different from Koren and Tenreyro (2013), as the crucial variable is the intensity of intermediate goods in production rather than the number of varieties of intermediate goods. Here the intensity of intermediate goods determines the effect of sectoral TFP shocks on aggregate TFP. These, as in standard business cycle models, are "aggregate" shocks, as it is not possible to insure against them. In Koren and Tenreyro (2013) instead, an increase in the number of varieties of intermediate goods in production is potentially able to reduce aggregate volatility, because varieties are subject to uncorrelated shocks. Thus, the two theories should be regarded as complementary as both present mechanisms that contribute to reduce aggregate volatility along the development path.

The remaining of the paper is as follows: section 2 reports the empirical evidence on the share of services, growth and volatility; section 3 presents the model while section 4 describes the quantitative results; finally, section 5 concludes.

## 2 Facts on Services Share, Growth and Volatility

In this section I provide empirical evidence on the relationship between the size of the services sector and growth and volatility. These empirical results motivate the theoretical model presented in the following section.

**Fact 1:** There is a negative relationship between the share of services in GDP and the growth rate of per-capita GDP for High Income and Upper Middle Income economies.

Consider the following random effects equation

\[
\gamma_{it} = \phi + \beta \text{Share}_{it} + \kappa_i + \varepsilon_{it},
\]

(1)
where $\gamma_{it}$ is country $i$ per-capita GDP growth rate in year $t$, $Share_{it}$ is the share of services in GDP at $t$, $\kappa_i$ is country $i$ random effect and $\varepsilon_{it}$ is the within country error. I estimate (1) for different groups of countries. Results are reported in table 1.\(^8\)

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<th>Table 1: Share of Services and GDP growth, 1961-2012</th>
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The first four groups in table 1 are those defined according to countries income level by the World Bank: High, Upper Middle, Lower Middle and Low income.\(^9\) There is a negative relationship between the share of services and per-capita GDP growth for High and Upper-Middle income economies. For both groups the relationship is statistically significant at 10% level. For Lower Middle and Low income economies both the constant term and the coefficient are not statistically significant. Thus, results in table 1 suggest that there is a negative relationship between the share of services and GDP growth after a certain level of income.

Consider now the estimate of (1) for the group of OECD economies. In this case the magnitude of the coefficient $\beta$ is similar to that obtained for the whole group of High income economies but the statistical significance of both the constant term and the coefficient increase. Further, consider the possibility that these results be driven by the large growth rates

\(^8\)Standard errors are heteroscedasticity robust. By using fixed-effects estimation results are very close to those in table 1. However, the Hausman test suggests to use random effects.

\(^9\)Refer to the data appendix for a description of the dataset. Country classification by the World Bank can be found at http://siteresources.worldbank.org/DATASTATISTICS/Resources/CLASS.XLS
of economies that experienced dramatic changes in policies and institutions (ex-communist countries) or by countries for which openness is commonly regarded as the main engine of growth (Ireland and South Korea). I thus estimate (1) for OECD economies by excluding the following countries: Czech Republic, Estonia, Ireland, Poland, Slovak Republic, Slovenia, South Korea. The negative relationship is maintained, while the significance of both the coefficient and the intercept is higher than for the whole group of OECD economies.

Finally, I estimate (1) for the groups of High and Upper Middle Income economies together. As expected, the negative relationship is maintained, but the significance of the coefficient is larger than when estimating (1) for each group.

Fact 2: The share of services in GDP and the volatility of per-capita GDP are negatively related.

Several papers document the negative relationship between volatility and development, where the proxy for the latter is usually the level of GDP per-capita. Here I provide evidence on the relationship between volatility and the size of the services sector. To do this I run the following regression

$$\text{sd}(\gamma_i) = \varphi + \lambda \text{Share}_{i1970} + \epsilon_{it},$$

where \(\text{sd}(\gamma_i)\) is the standard deviation of per-capita GDP growth during the 1970-2012 period in country \(i\) and \(\text{Share}_{i1970}\) is the share of services in country \(i\) in 1970. Results are reported in table 2. The relationship is negative and significant. As a robustness check I also report the same estimation by substituting the share in 1970 with those at the beginning of each subsequent decade. The results show that the relationship is remarkably stable.

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11 This is the same approach as in Acemoglu and Zilibotti (1997), replacing the beginning of sample level of per-capita GDP with the beginning of sample share of services.

12 To compute standard deviations of GDP growth in tables 2 and 3 I use the period 1970-2012 instead of 1961-2012 as in table 1. This is because the share of services in 1961 is reported by the World Bank for a very small number of countries. By regressing standard deviations computed for the period 1961-2012 on services share in 1970 (and subsequent decades) the results are very similar to those reported in tables 2 and 3.
I also estimate (2) for the group of High and Upper Middle income economies, for which a negative relationship between growth and services share was found above. The results are reported in table 3. The negative relationship is maintained and it is similar in magnitude to that in table 2. Also in this case the relationship appears to be persistent over time. Only for the share in 2010 the estimated coefficient is not significant.
**Fact 3:** *The share of services in GDP and per-capita GDP are positively related.*

Several papers document the correlation of the share of services with per-capita income, e.g. Echevarria (1997), Buera and Kaboski (2012) and Herrendorf, Rogerson and Valentinyi (2013) among many others. Here I am interested in testing this relationship for the group of countries for which a statistically significant relationship between the services share and both growth and volatility has been found (fact 1). Thus, I run a panel regression between the share of services and GDP per-capita for the groups High and Upper Middle income economies:

$$Share_{it} = \xi + \eta y_{it} + v_i + \xi_{it},$$

where $y_{it}$ is per-capita GDP in country $i$ at $t$, $v_i$ is country $i$ random effect and $\xi_{it}$ is the within country error. As reported in table 4, the relationship is positive for both income groups and for the two groups as a whole.

| Table 4: Per-capita GDP and Services Share (HI and UMI), 1961-2012 |
|--------------------|--------------------|-------------------|---------------|----------------|---------------|
|                    | Serv. Sh | Coef.      | Std. Err.  | P>|z| | Countries | Observations |
| **Het. Rob.**       |          |            |              |              |              |                |                |
| HI                  | Per-capita GDP | 0.00049 | 0.00010 | 0.000 | 53 | 1750          |
| constant            |          | 50.42     | 2.31        | 0.000      |              |                |                |
| UMI                 | Per-capita GDP | 0.00318 | 0.00063 | 0.000 | 53 | 1811          |
| constant            |          | 41.92     | 2.93        | 0.000      |              |                |                |
| HI and UMI          | Per-capita GDP | 0.00056 | 0.00011 | 0.000 | 106 | 3561          |
| constant            |          | 50.26     | 1.65        | 0.000      |              |                |                |
3 The Model

I consider a model in which structural change is driven by standard non-homothetic preferences of the Stone-Geary type, as in Kongsamut, Rebelo and Xie (2001). The setting is similar to Moro (2012).

3.1 Firms

There are two sectors in the economy, manufacturing and services. The representative firm in each sector produces gross output using a Cobb-Douglas production function in labor, manufactured intermediate goods and intermediate services. I abstract from capital for simplicity here. In subsection 3.4 below I discuss the implications of allowing for capital accumulation.

The gross output production function of the representative firm in the manufacturing sector is

\[ G_m = B_m N_m^\nu_m (M_m^{\varepsilon_m} S_m^{1-\varepsilon_m})^{1-\nu_m}, \]  

and that of the representative firm in the services sector is

\[ G_s = B_s N_s^\nu_s (M_s^{1-\varepsilon_s} S_s^{\varepsilon_s})^{1-\nu_s}, \]

where \(0 < \nu_j < 1, 0 < \varepsilon_j < 1\), \(N_j\) is labor, \(M_j\) is the manufactured intermediate good, \(S_j\) is intermediate services and \(B_j\) is gross output TFP, with \(j = m, s\).\(^{13}\) Gross output TFP of each sector follows a stochastic process, unspecified for the time being. In each sector, the representative firm maximizes profits by taking as given the price of the manufacturing good \(p_m\), the price of services \(p_s\) and the wage rate \(w\).\(^{14}\)

The input-output structure of the economy, together with competitive markets and constant returns to scale in production implies that the relative price of services with respect to manufacturing, \(p_s/p_m\), is independent of the quantities produced of the two goods. This is given by

\[ \frac{p_s}{p_m} = \Omega(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) \left( \frac{B_m}{B_s} \right)^{\frac{\nu_m}{\nu_m [1-\varepsilon_m (1-\varepsilon_s)] + \nu_s [1-\varepsilon_m (1-\varepsilon_s)] - \nu_s \nu_m}} = \frac{1}{\nu_s [1-\varepsilon_m (1-\varepsilon_s)] - \nu_s \nu_m}. \]  

\(^{13}\)In Moro (2012) I show that the share of intermediate goods in the gross output production of manufacturing and services does not show long run trends in the U.S. Similar evidence is presented in Herrendorf, Herrington and Valentinyi (2013) who support the use of a Cobb-Douglas gross output production function.

\(^{14}\)See appendix A for an explicit formulation of the firm problem.
Details of the derivation are given in appendix A. In (5), $\Omega$ is a function of the parameters $\nu_m$, $\nu_s$, $\varepsilon_m$ and $\varepsilon_s$. Thus, the relative price of the two goods is technologically determined as it depends on the parameters of the production functions and on gross output TFP, $B_m$ and $B_s$. This result follows from the non-substitution theorem (Samuelson, 1951).

### 3.2 Households

The model economy is inhabited by a measure one of identical households, indexed by $i$ in the interval $[0, 1]$. Households in this economy have preferences over manufacturing and services consumption and are endowed with one unit of labor services each period.

The utility function of the representative household at date $t$ is given by

$$u = \log \left[ bc_m^p + (1 - b) (c_s + \bar{s})^p \right]^{\frac{1}{p}} + \varphi \log(1 - n),$$

(6)

where $c_m$ and $c_s$ are the per-capita consumption levels of manufacturing and services and $n$ are per-capita labor services.\(^{15}\) In (6), $\rho \leq 1$, $b \in [0, 1]$, $\bar{s} > 0$ and $\varphi > 0$. As in Kongsamut, Rebelo and Xie (2001), $\bar{s}$ is interpreted as home production of services. Note that the utility function in (6) is appropriate to study GDP growth and volatility in a multi-sector model following the results in Herrendorf, Rogerson and Valentinyi (2013), who show that Stone-Geary preferences provide a good fit of post-war expenditure shares in the U.S. Once the consumption index is defined as

$$c = \left[ bc_m^p + (1 - b) (c_s + \bar{s})^p \right]^{\frac{1}{p}},$$

the utility function in (6) coincides with the one often used in growth theory and in the real business cycle literature.

Each period, the household decides the amount of labor services to supply, earns a total wage $wn$, and spends it in manufacturing and services consumption. Thus, the problem of each household at time $t$ is to maximize (6) subject to the budget constraint $p_sc_s + p_mc_m = wn$. The first-order conditions for the household problem deliver

$$c_m = \frac{(w/p_m)n + (p_s/p_m)\bar{s}}{(p_s/p_m)^{\frac{1}{1-\rho}} (\frac{1-b}{b})^{\frac{1}{1-\rho}} + 1},$$

(7)

$$c_s = \frac{(w/p_m)n + (p_s/p_m)\bar{s}}{(p_s/p_m)^{\frac{1}{1-\rho}} (\frac{b}{1-b})^{\frac{1}{1-\rho}} + p_s/p_m} - \bar{s},$$

(8)

\(^{15}\)As households are identical I avoid the use of the index $i$. 

12
and
\[
\frac{bc_m^{p-1}(1 - n)}{bc_m^p + (1 - b)(c_s + s)^p} = \frac{\varphi p_m}{w}.
\]
In equilibrium, the wage rate (in manufacturing units) \( w/p_m \) and the relative price \( p_s/p_m \) are uniquely determined by gross output TFP levels and the elasticities of the production functions, and the three first-order conditions above allow to solve for \( c_m, c_s \) and \( n \).

### 3.3 The Competitive Equilibrium

A competitive equilibrium for this economy is a set of prices \( \{p_s, p_m, w\} \), allocations for the households \( \{c_s, c_m, n\} \), for the manufacturing firm \( \{N_m, M_m, S_m\} \) and for the services firm \( \{N_s, M_s, S_s\} \) such that, given prices: a) \( \{c_s, c_m, n\} \) solve the household problem; b) \( \{N_m, M_m, S_m\} \) solve the manufacturing firm problem; c) \( \{N_s, M_s, S_s\} \) solve the services firm problem; and d) markets clear:

\[
G_m = \int_0^1 c_m \, di + M_m + M_s = c_m + M_m + M_s,
\]
\[
G_s = \int_0^1 c_s \, di + S_m + S_s = c_s + S_m + S_s,
\]
\[
\int_0^1 ndi = n = N_m + N_s.
\]

### 3.4 Implications of the theoretical framework

In this subsection I study the relationship between the relative size of the two sectors and the growth and volatility performance of aggregate output in the model economy. To do this, I characterize the production possibility frontier of the economy. Each point on the frontier represents a different relative size of the two sectors, thus by studying the performance of the economy along the frontier it is possible to assess the role of the structural composition in shaping aggregate growth and volatility. To provide intuition I will first illustrate the effect of the production structure on aggregate TFP growth and volatility by focusing on the two extreme points of the production possibility frontier. These points are those of complete specialization, in which the economy consumes only manufacturing in one case and only services in the other. Next, I will characterize growth and volatility of aggregate output along the entire frontier in the special case of Cobb-Douglas preferences, for which analytical expressions can be obtained.
To find the production possibility frontier at a given point in time it is sufficient to solve the following problem

\[
\max_{N_m, M_m, S_m, M_s, S_s} \left[ B_m N_m^{\nu_m} \left( M_m^{\epsilon_m} S_m^{1-\epsilon_m} \right)^{1-\nu_m} - M_m - M_s \right]
\]  \hspace{1cm} (9)

subject to

\[
B_s (n - N_m)^{\nu_s} \left( M_s^{1-\epsilon_s} S_s^{\epsilon_s} \right)^{1-\nu_s} = S_m + S_s + c_s,
\]

where \(B_m N_m^{\nu_m} \left( M_m^{\epsilon_m} S_m^{1-\epsilon_m} \right)^{1-\nu_m}\) and \(B_s (n - N_m)^{\nu_s} \left( M_s^{1-\epsilon_s} S_s^{\epsilon_s} \right)^{1-\nu_s}\) are the gross output production functions defined in (3) and (4) and \(n\) is the total amount of labor used in production in the economy in the period considered. The solution to problem (9) determines the maximum amount of manufacturing that can be consumed in the economy given labor services \(n\) and an amount \(c_s\) of services consumption. When \(c_s = 0\), the solution to (9) determines the point in which the production possibility frontier of this economy crosses the manufacturing axis. Note also that when \(c_s = 0\) the constraint in (9) implies that the services sector becomes purely an intermediate sector as it produces only intermediate services used in the production of manufacturing and of services themselves.

The solution to problem (9) at time \(t\) for \(c_s = 0\) is

\[
V_{m,t} = \Theta_m(\nu_m, \nu_s, \epsilon_m, \epsilon_s)B_{m,t}^{f_1(\nu_m, \nu_s, \epsilon_m, \epsilon_s)}B_{s,t}^{f_2(\nu_m, \nu_s, \epsilon_m, \epsilon_s)}n_t,
\]  \hspace{1cm} (10)

where \(\Theta_m, f_1\) and \(f_2\) are functions of \(\nu_m, \nu_s, \epsilon_m\) and \(\epsilon_s\). Note that competitive markets, constant returns to scale in production and the input-output structure of the economy imply that the production possibility frontier of this economy is linear.\(^{16}\) Thus, \(p_s/p_m\) in (5) gives the feasible amount of manufacturing that can be consumed in the economy by reducing the consumption of services by one unit. It follows that by dividing (10) by (5) it is possible to derive the maximum amount of services that can be consumed when the manufacturing sector produces only intermediate goods

\[
V_{s,t} = \Theta_s(\nu_m, \nu_s, \epsilon_m, \epsilon_s)B_{m,t}^{f_3(\nu_m, \nu_s, \epsilon_m, \epsilon_s)}B_{s,t}^{f_4(\nu_m, \nu_s, \epsilon_m, \epsilon_s)}n_t,
\]  \hspace{1cm} (11)

where \(\Theta_s, f_3\) and \(f_4\) are also functions of \(\nu_m, \nu_s, \epsilon_m\) and \(\epsilon_s\).\(^{17}\) Expressions (10) and (11) represent the economy’s output in two extreme cases, one in which only manufacturing is

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\(^{16}\) The linearity of the production possibility frontier comes from the fact labor is the only primary input. Thus, value added production functions in manufacturing and services are both linear in labor, implying the linearity of the frontier. See appendix C for a definition of the value added production functions.

\(^{17}\) Details of the derivation and the explicit functional form of \(\Theta_m, \Theta_s, f_1, f_2, f_3\) and \(f_4\) are given in Appendix A.
consumed and services is only an intermediate sector, and another in which the opposite situation holds. Thus, (10) can also be interpreted as the aggregate production function of an economy consuming only manufacturing and (11) the corresponding function of an economy consuming only services. In this view, the difference between (10) and (11) lies in the TFP term that multiplies aggregate labor services. From (10), aggregate TFP when the economy produces only manufacturing is

\[ TFP_{m,t} = \Theta_mB_{m,t}^f B_{s,t}^f. \] (12)

Instead, when the economy produces only services, aggregate TFP is, from (11),

\[ TFP_{s,t} = \Theta_sB_{m,t}^{f_3} B_{s,t}^{f_4}. \] (13)

Thus, for given processes of \( B_{m,t} \) and \( B_{s,t} \), the pattern of the aggregate TFP term depends on the value of \( f_1 \) and \( f_2 \) when the economy produces only manufacturing, and of \( f_3 \) and \( f_4 \) when the economy produces only services. In what follows, I analyze the particular case in which gross output TFP in manufacturing and services grows following a common stochastic growth factor.

**Assumption 1.** Let gross output TFP in manufacturing and services evolve at time \( t \) according to a common growth factor

\[ \frac{B_{m,t}}{B_{m,t-1}} = \frac{B_{s,t}}{B_{s,t-1}} = (1 + \gamma_B) e^{z_t}, \] (14)

where \( \gamma_B > 0 \) and \( z_t \) is a random component with zero mean and finite variance.

By assuming a common process for technological change in the two sectors, it is possible to study the role of the different production technologies in manufacturing and services on aggregate TFP growth and volatility. In particular, when (14) holds, differences in growth and volatility between (12) and (13) are uniquely determined by differences in the elasticity of output with respect to inputs in the two sectors, which determine the values of \( f_1 \), \( f_2 \), \( f_3 \) and \( f_4 \). Thus, before stating proposition 1, which characterizes the relationship between the growth rates of (12) and (13), I make the following assumption on the intensity of intermediate goods in the two sectors:

**Assumption 2.** Let \( 1 - \nu_m > 1 - \nu_s \).

Assumption 2 imposes that the elasticity of output with respect to intermediates be larger in manufacturing than in services. The next proposition characterizes the relationship between the growth rate of \( TFP_{m,t} \), \( \gamma_{tfp,m} \), and that of \( TFP_{s,t} \), \( \gamma_{tfp,s} \).
Proposition 1. Let assumption 1 and 2 hold and the variance of $z_i$ be zero (deterministic growth). Then,

$$\gamma_{\text{tfp,m}} > \gamma_{\text{tfp,s}},$$

aggregate TFP growth is larger when the economy consumes only manufacturing every period than when it consumes only services every period.

Proof. See Appendix B. □

Similarly, the following proposition states an equivalent result for the relationship between
the volatility of (12) and (13), measured as the standard deviation of growth rates.

Proposition 2. Let assumption 1 and 2 hold. Then,

$$sd(\gamma_{\text{tfp,m}}) > sd(\gamma_{\text{tfp,s}}),$$

aggregate TFP volatility is larger when the economy consumes only manufacturing every period than when it consumes only services every period.

Proof. See Appendix B. □

Proposition 1 and 2 state that, for a common pattern of gross output TFP in the two sectors, when production of manufacturing is more intensive in intermediate goods than production of services, the economy displays a larger aggregate TFP growth and volatility in the case it consumes only manufacturing with respect to the case in which it consumes only services. To see the intuition for this result, note first that in models with input-output linkages intermediate goods provide an amplification mechanism on the effect of sectoral TFP changes on aggregate TFP. Consider for instance a 1% TFP increase in both sectors in an economy governed by a benevolent social planner. For sake of intuition assume also that the amount of labor used in each sector is given. With the same amount of labor and intermediates as before, sectors can now produce a 1% larger amount of gross output. Assume that the planner allocates the gross output of each sector in given proportions to final demand (i.e aggregate output) and to intermediate goods provision. Thus, the 1% increase in gross output TFP implies a 1% increase in aggregate output (which raises aggregate TFP by 1% as total labor is constant) and a 1% increase in intermediate goods provision in the economy. These additional intermediates allow a further increase in sectoral gross output and, in turn, in aggregate TFP. Thus, the initial 1% increase of gross output TFP is amplified at the aggregate level through intersectoral linkages and implies a final effect on aggregate
TFP larger than 1%\textsuperscript{18}. Put it differently, an increase in gross output TFP, not only makes sectors more productive, but also provides the economy with more intermediates available for production. As a result aggregate TFP raises both because sectoral productivity is larger and because there is a larger amount of intermediates in the economy.

In turn, the strength of the amplification mechanism depends on the intensity of intermediate goods in production, which determines the increase in gross output for an additional unit of intermediates. When the two sectors in the economy display a different intensity of intermediate goods in production, the magnitude of the amplification mechanism becomes endogenous to the structure of the economy. In particular, this magnitude increases with the size of the sector with the largest intensity of intermediate goods in production. As a result, if manufacturing displays a larger intensity of intermediate goods with respect to services, a common TFP increase in the two sectors implies a larger growth of aggregate TFP when the economy produces only manufacturing than when it produces only services. An equivalent reasoning holds for volatility.

In the equilibrium of the model, the intensity of intermediate goods in production is equal to the share of intermediate goods in gross output of the sector considered. Thus, using data on the share of intermediate goods in the two sectors it is possible to assess whether assumption 2 holds empirically. Figure 1 shows that this is the case for a group of 26 developing and developed countries. The average share of intermediate goods in manufacturing gross output is 0.64, while it is 0.40 in services. Also, figure 1 suggests that the intensity of intermediate goods in each sector is similar across developed and developing economies. Thus, if output elasticities in the two sectors are calibrated using data from any of the 26 countries in figure 1, assumption 2 is satisfied.

As in standard exogenous growth models and RBC models, aggregate TFP here is the driving force behind output movements. Thus, by affecting aggregate TFP, the structure of the economy drives the behavior of aggregate output. However, with non-homothetic preferences such as those in (6), the structure of the economy also affects labor supply, and an explicit relationship between sectors’ relative size and output cannot be derived. In what follows I derive such relationship for the special case of a Cobb-Douglas consumption

\textsuperscript{18}This result holds for a wide variety of sectoral technologies. Necessary conditions for the mechanism to work are constant returns to scale in production and quasi-concave and continuously differentiable technologies. Formal proof is given in Hulten (1978).
index, which implies a given structure of the economy. This case is made in the following assumption.

**Assumption 3.** Assume that $\rho = \bar{s} = 0$ and that aggregate output is defined by $y = c$.

Assumption 3 implies that the utility function (6) reduces to $u = \log \left( c_m c_s^{1-b} \right) + \varphi \log (1-n)$, the consumption index is given by the Cobb-Douglas function $c = c^b_m c_s^{1-b}$, and sectors’ size relative to output is fixed and given by $b$ for manufacturing and $1-b$ for services. Also, with no investment in the economy, the natural definition of aggregate output is $y = c$. The following proposition characterizes the relationship between output growth and sectors’ size.

**Proposition 3.** Let assumptions 1, 2 and 3 hold and the variance of $z_t$ be zero (deterministic growth). Then, the output growth rate is given by

$$
\gamma_y = b \gamma_{tp,m} + (1-b) \gamma_{tp,s},
$$

and the larger is $b$, the larger are output growth and aggregate TFP growth.

Proof. See Appendix B. ■

The following proposition states an equivalent result for output volatility.
Proposition 4. Let assumptions 1, 2 and 3 hold. Then,

$$\text{sd}(\gamma_y) = [b(f_1 + f_2) + (1 - b)(f_3 + f_4)] \text{sd}(z_t)$$

and the larger is $b$, the larger are output volatility and aggregate TFP volatility.

Proof. See Appendix B.

Propositions 3 and 4 state that the larger the relative size of manufacturing (which displays the largest intensity of intermediates in production), the higher the growth rate and volatility of output and aggregate TFP. The intuition is the same as the one given for propositions 1 and 2. Intermediate goods provide an amplification mechanism which is endogenous to the structure of the economy when the two sectors display a different intensity of intermediates in production. Note that propositions 3 and 4 include the cases in which only manufacturing is consumed, $b = 1$, and in which only services is consumed, $b = 0$. Thus, aggregate output growth and volatility are driven by aggregate TFP growth and volatility which, in turn, depend on the structure of the economy.

The analysis in this subsection describes the relationship of output growth and volatility with the structure of the economy under assumptions 1, 2 and 3. In the next section I use the model to quantitatively assess the importance of the structural composition for the observed differences in GDP growth and volatility between Upper-Middle and High income economies. To do this I drop assumptions 1 and 3 of the current subsection. First, I abandon assumption 3 to study a model economy in which the relative size of services endogenously increases over time as gross output TFP grows in the two sectors. In this case, the effect on output of gross output TFP movements depends both on the structure of the economy and on the speed of structural change (which in turn depends on the parameters $\rho$ and $\bar{s}$), and the equivalent to propositions 3 and 4 cannot be derived. However, the calibrated version of the model used in the next section provides quantitative results suggesting that the negative relationship of output growth and volatility with the relative size of the services sector still holds with non-homotheticity of preferences and an elasticity of substitution between goods in consumption different from one. Next, I also drop assumption 1 by calibrating gross output TFP processes to the U.S. experience. The result is that the effect of structural change on growth and volatility is similar to the case in which a common gross output TFP process is assumed.

Note here that introducing capital accumulation would not affect the predictions of the
model. This is because the main mechanism works through the effect of structural change on aggregate TFP. In a model with capital accumulation decisions a decline in aggregate TFP growth due to the structural change from manufacturing to services implies a lower growth rate of GDP and lower investment rates (Echevarria, 1997) while a decline in aggregate TFP volatility (due either to a structural change or to an exogenous change) implies a lower volatility of GDP and investment (Moro, 2012). Thus, it is possible that the effect of structural change in a model with capital is magnified through the response of investment to the change in aggregate TFP behavior. However, introducing capital accumulation in the quantitative exercises of the next section would require to solve numerically for a stochastic and non-balanced growth path, which poses a heavy computational burden. For this reason, and given the reasonably good performance of the model in the quantitative exercises, capital accumulation is not considered here.

4 Quantitative analysis

4.1 Strategy

In this section I describe the strategy adopted to compare model and data to quantify the importance of the size of the services sector in shaping per-capita GDP growth and volatility along the development path. The first step is to construct a statistic for growth and volatility in the data that can be compared to the model’s output. As noted in Lucas (1988), the cross country variability in growth rates is high for middle income economies while it is low for high income economies. Thus, the comparison of per-capita GDP growth and volatility between a high income and a middle income economy crucially depends on the middle income country chosen, even when controlling for the share of services in GDP. To deal with this issue, one way to proceed is to compare the two groups of countries defined High Income (HI) and Upper Middle Income (UMI) by the Word Bank. For these two groups of economies, the econometric analysis in section 2 suggests that there is statistically significant relationship between the share of services in GDP and per-capita GDP growth and volatility. Figure 2 reports the average share of services in GDP for selected groups of countries within the two categories.\textsuperscript{19} The share of services of UMI increases from 0.46 to 0.56 in the 1970-2010

\textsuperscript{19}To construct figure 2, I use all countries for which the share of services and the growth rate of per-capita GDP are reported from 1970 to 2010 in the World Development Indicators. These are 15 Upper Middle
period, while that of HI increases from 0.54 to 0.69. During the same period, the UMI GDP per-capita grows at 2.57% per-year, while the HI one at 2.30%. Volatility, measured as the standard deviation of per-capita GDP growth rates over the period, is 3.82% for UMI and 3.06% for HI.\textsuperscript{20} Thus, the average difference between the two groups is 0.27\% in terms of growth and 0.76\% in terms of volatility.

An alternative way to construct data statistics comparable to the model’s outcome is to use the panel of countries employed to construct figure 2 to estimate a random effect equation as in (1). There are two advantages in doing this. First, the panel estimation controls for countries idiosyncratic conditions. Second, once all HI and UMI countries are pooled together, this methodology provides a unique statistic relating the share of services and GDP growth over the development path, regardless of the distinction middle/high income. By estimating (1) using the balanced panel of countries employed to construct figure 2, I obtain a coefficient $\beta = -0.093$ (s.e. 0.018). This implies that following an increase in the share of services of $\Delta Share_{it}$, the growth rate of an economy should decline by $\Delta \gamma_{it} = -0.093 \Delta Share_{it}$. The average share of services in the period 1970-2010 for the group of UMI economies in figure 2 is 0.50 while that of HI economies is 0.62. Thus, the difference in growth implied by the panel estimation is $\Delta \gamma_{it} = -0.093 \times 0.12 = -1.12\%$.\textsuperscript{21} In

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Share of Services in GDP in Upper-Middle income and High income countries from 1970 to 2010. Source: World Bank.}
\end{figure}

\textsuperscript{20}GDP is measured in 2000 U.S. dollars.
\textsuperscript{21}By using fixed-effects estimation the coefficient $\beta$ becomes -0.108 (s.e. 0.031). However the Hausman
this case, the differences in growth between UMI and HI countries is more than four times larger than when considering simple averages within the two group of countries (-0.27%).

A similar argument can be used to obtain a statistic relating the share of services and per-capita GDP volatility in the data. By considering the balanced panel of countries used to construct figure 2, I run a regression between the average share of services during the period 1970-2010 and volatility during the same period. The estimated equation is

\[ sd_{70,10}(\gamma_i) = \varphi + \lambda \text{mean}_{70,10}(\text{Share}_i) + \epsilon_{it}, \]  

(15)

where \( sd_{70,10}(\gamma_i) \) is the standard deviation of per-capita GDP growth during the period 1970-2010 for country \( i \), and \( \text{mean}_{70,10}(\text{Share}_i) \) is the average share of services of country \( i \) during the same period. The estimated coefficient is \( \lambda = -0.074 \) (s.e. 0.017), which implies that a country with an average share of services of 0.62 during the period has a volatility \( \Delta sd_{70,10}(\gamma_i) = -0.074 \times 0.12 = -0.89\% \) smaller than one with an average share of 0.50 in the same period. In this case the difference in volatility between UMI and HI countries is close to that obtained using simple averages (-0.76%).

To isolate the effect of structural change on GDP growth and volatility in the model I first study the case in which manufacturing and services gross output TFP is driven by a common process. This way, the model can be used to address the following question: if gross output TFP processes in the two sectors were to be generated by the same stochastic process, how much of the difference in growth and volatility observed in the data between UMI and HI economies can be accounted for by structural change? To address this issue, I proceed as follows. As one period in the model is one year in the data, I calibrate the model such that, given a common TFP process in the two sectors, it generates an increase in the share of services in GDP from 0.46 to 0.69 in 82 periods. Also, the calibration requires that in the first 41 periods GDP growth and volatility match those of UMI economies in the 1970-2010 period. In this way, the calibrated model generates a transition path that matches GDP growth and volatility of UMI economies in the first half of the transition. Next, by measuring GDP growth and volatility generated by the model in the last 41 periods, and comparing them with the corresponding figures for HI economies, it is possible to quantify the effect of structural change on growth and volatility.

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test suggests to use random effects.
Next, I study the performance of the model when TFP processes in the two sectors are calibrated to the U.S., a country that experienced a large process of structural transformation. With respect to the previous quantitative exercise, this one has the advantage of not imposing any restriction on growth and volatility in the first part of the transition path. Thus, the levels of growth and volatility displayed by the model along the transition path are endogenously determined given TFP process of the U.S. economy and the calibrated transition path of the share of services.

4.2 Calibration

To simulate the model it is necessary to calibrate eight parameters and the gross output TFP process in the two sectors. Calibration of the technology parameters $\nu_m$, $\nu_s$, $\varepsilon_m$ and $\varepsilon_s$ requires data on the value of gross output, labor and intermediate inputs for the manufacturing and the services sector. These data are not available for the two groups of HI and UMI countries in the World Bank dataset. However, the analysis in the previous section suggests that the relevant condition for the decline in aggregate TFP growth and volatility when the share of services in GDP increases is $1 - \nu_m > 1 - \nu_s$, which is true for all countries in figure 1. In addition, the absolute values of $1 - \nu_m$ and $1 - \nu_s$ are roughly constant across countries. Thus, I calibrate $\nu_m$, $\nu_s$, $\varepsilon_m$ and $\varepsilon_s$ using U.S. data from Jorgenson dataset, 2007. The Cobb-Douglas assumption implies that the elasticities of output with respect to inputs are equal to the inputs shares of gross output in equilibrium. Parameters are then set equal to the average shares during the 1960-2005 period. I obtain $\nu_m = 0.32$, $\nu_s = 0.51$, $\varepsilon_m = 0.71$ and $\varepsilon_s = 0.72$. As in Duarte and Restuccia (2010), I normalize to one sectoral TFP levels in the first period in both sectors, $B_{m,1} = B_{s,1} = 1$. On the demand side, I set $\rho$ according to Rogerson (2008) and Duarte and Restuccia (2010), equal to $-1.5$.

Note that, consistently with Jorgenson dataset, in this paper $n$ denotes labor services. Growth in labor services is computed in the data as a weighted average of growth in hours worked of several types of labor, where weights are given by the share of each type of labor

\[\text{Note that in national accounts, gross output sums to rents accruing to capital, labor and intermediates. In the model capital is not modeled so it is necessary to attribute capital rents to the other factors. In the calibration in the text, these rents are attributed proportionally to labor and intermediate goods. In figure 1 instead, the share of intermediate goods is computed by taking for each sector the ratio of the value of intermediates over the value of gross output.}\]
in total labor compensation.\textsuperscript{23} Thus, when analyzing long time spans, labor services appear as a more appropriate measure of the labor input than other measures such as hours or employment, because they take into account changes in the composition of labor that are not to be attributed to gross output TFP measures. In the model, given the utility function (6), the amount of labor services in equilibrium is given by

\[ n = \frac{1}{1 + \varphi} \left( 1 - \frac{\varphi \bar{s}}{\Theta s B^f m B^f s} \right), \quad (16) \]

which implies that when gross output TFP grows (in any sector), equilibrium labor increases.\textsuperscript{24} This result is due to the non-homotheticity parameter \( \bar{s} \). When the latter is zero labor is constant in equilibrium for any level of sectoral TFP and equal to \( 1/(1 + \varphi) \).\textsuperscript{25}

Thus, the remaining three preference parameters, \( \bar{s}, b \) and \( \varphi \), and the common sectoral TFP growth rate in the two sectors, \( \gamma_B \), are set to match the following four targets in the data. Two targets are the share of services in GDP in the first period (0.46), which corresponds to that of UMI economies in 1970, and that in the last period (0.69), which is the share in HI economies in 2010. The third target is the increase in per-capita labor services over time. This target is not available for the two groups of HI and UMI economies. I measure the total growth in per-capita labor services in the U.S. during the 1960-2005 period using Jorgenson dataset and find an average yearly growth rate of 0.43%. Using this measure, the third target of the calibration is an increase in per-capita labor services of a factor \( (1 + 0.0043)^{81} = 1.4156 \) between periods 1 and 82. The fourth target is an average growth rate of GDP equal to 2.57% during the first 41 periods, which is that of UMI economies in the 1970-2010 period. The calibrated parameters are \( \bar{s} = 0.0054, b = 0.0594, \varphi = 7.8143, \gamma_B = 0.0082 \).\textsuperscript{26} To interpret the calibrated value of \( \bar{s} \), note that this implies a

\begin{itemize}
  \item See Jorgenson, Gollop and Fraumeni (1987) and O’Mahony and Timmer (2009) for a detailed description of the methodology used to construct series of labor services.
  \item In the literature on structural change, labor is not usually introduced in the utility function. See for instance Echevarria (1997), Ngai and Pissarides (2007) or Duarte and Restuccia (2010). The main reason is that these papers do not address quantitative business cycle issues. In this paper instead, endogenous labor is crucial for the model to quantitatively come to terms with volatility data. Other papers that introduce labor into the utility function in a model of structural transformation are Da-Rocha and Restuccia (2006) and Moro (2012). The business cycle analysis in these contributions is performed by comparing GDP volatility across steady states with a different share of services in GDP. In contrast, here I study business cycle properties along a transition path in which the share of services in GDP continuously increases as GDP grows.
  \item Note that labor services tend to the constant value \( 1/(1 + \varphi) \) as income increases. This is because, as \( B^m \) and \( B^s \) grow, the term \( (\varphi \bar{s}) / (\Theta s B^f m B^f s) \) tends to zero.
  \item As discussed in the text, (16) implies that for any positive value of \( \varphi \), labor increases in the model as
\end{itemize}
share of market consumption in total services consumption, $c_s/(c_s + \bar{s})$, of 43% in period one and of 87% in period 82.

Finally, I need to calibrate the stochastic process for gross output TFP shocks. These evolve according to

$$B_{j,t} = B_{j,t-1}(1 + \gamma_B) e^{z_j,t},$$  \hspace{1cm} (17)

with $j = m, s$, $z_{j,t} = \rho_z z_{j,t-1} + \epsilon_{j,t}$, $\epsilon_{j,t} \sim N(0, \sigma^2)$ and i.i.d. over time. Equation (17) states that in each sector sectoral TFP grows at the constant rate $\gamma_B$ and receives a shock $z_{j,t}$ at each $t$. The shock $z_{j,t}$ follows the same stochastic process in both sectors. If shocks are set to zero, $z_{j,t} = 0 \forall j, t$, gross output TFP grows at the deterministic rate $\gamma_B = 0.0083$.

To calibrate $\rho_z$ I use data from the U.S. manufacturing and services sectors in Jorgenson dataset. I first compute series for $B_{m,t}$ and $B_{s,t}$ using the production functions (3) and (4) and data of sectoral gross output, labor and intermediate goods in manufacturing and services. Then, I log and detrend the series $B_{m,t}$ and $B_{s,t}$ with an Hodrick-Prescott filter, and estimate an AR(1) process of the percentage deviations. The estimated coefficients are 0.6249 for services and 0.6293 for manufacturing. Thus, I set $\rho_z = 0.63$ for both sectors. Finally, I set $\sigma = 0.0136$, so that the model matches a standard deviation of GDP growth rates equal to 3.82% in the first 41 periods, which is that of UMI economies in the 1970-2010 period. Parameter values are reported in Table 5.

---

GDP grows. This in turn requires that to calibrate $\varphi$ labor has to display an increasing pattern over time in the data. Apart from labor services, measures of labor such as employment display an upward trend over time (in the U.S.) while other measures such as hours do not (average hours worked slightly fell from 1950 to 2000 in the U.S. [Duarte and Restuccia, 2007]). Calibrating the model to display a constant amount of labor over time would require to set $\varphi = 0$, which amounts to have exogenous labor, as in most models of structural change in the literature. However, as discussed in note 19, it is important to have an endogenous labor decision in the model when studying volatility dynamics.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_m$</td>
<td>Share of $N_m$ in $G_m$</td>
<td>0.32</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>Share of $M$ in manufac. interm.</td>
<td>0.71</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>Share of $N_s$ in $G_s$</td>
<td>0.51</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Share of $S$ in services interm.</td>
<td>0.72</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity parameter in preferences</td>
<td>-1.5</td>
</tr>
<tr>
<td>$b$</td>
<td>Weight of manufacturing in preferences</td>
<td>0.0594</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>Home production of services</td>
<td>0.0054</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Wedge between consumption and labor</td>
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</tr>
<tr>
<td>$B_{m,1}$</td>
<td>Manufacturing TFP level in the first period</td>
<td>1</td>
</tr>
<tr>
<td>$B_{s,1}$</td>
<td>Services TFP level in the first period</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma_B$</td>
<td>Average growth rate of TFP</td>
<td>0.0082</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Autoregressive Parameter</td>
<td>0.63</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of shocks</td>
<td>0.0136</td>
</tr>
</tbody>
</table>

### 4.3 Results

The first row of table 2 reports the average per-capita GDP growth rate and the standard deviation of per-capita GDP growth rates for the first 41 model periods and two calibration targets, the per-capita GDP growth rate and the standard deviation of per-capita GDP growth of UMI economies during the 1970-2010 period. The second row reports the same statistics for the last 41 model periods and for HI economies during the 1970-2010 period. All results are averages of 100,000 simulations. The third row of table 2 reports the ratio between the two cases. The average growth rate of per-capita GDP is 18% larger in the first 41 periods with respect to the last 41, compared to a difference in the data of 12%. Thus, structural transformation in the model generates a difference in growth rates larger than that observed in the data. Instead, when we consider the difference in growth between UMI and HI implied by the panel estimation, the model accounts for almost a half of it (18% versus 42%).

The difference in volatility is 16% in the model compared to 25% in the data. Thus, the different size of the services sector explains 64% of the difference in volatility between UMI and HI economies. Instead, the model accounts for roughly half of the differences in volatility between UMI and HI implied by the regression in (15) (16% versus 30%). Thus, the structural transformation in the model accounts for between 40% and 100% of decline.

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27 Aggregate real value added at $t$, which is the model’s counterpart of real GDP in the data, is computed as a chain-weighted Fisher index of sectoral value added. See appendix C for details.
in growth and for between 50% and 64% of the decline in volatility between UMI and HI income countries.

Consider now the change in volatility *within sectors* along the transition path. Koren and Tenreyro (2007 and 2013) document that a large part of the volatility decline observed along the development process is due to a reduction of volatility within sectors. This is consistent with the last two columns of table 6: the ratio of volatility between UMI and HI economies is 1.19 in manufacturing and 1.41 in services. The seventh and eighth columns of table 6 report the volatility of manufacturing and services real value added in the model in the two subperiods. While the volatility of manufacturing remains unchanged, the volatility of services is 31% larger in the first subperiod with respect to the second. Thus, the model can account for 76% of the decline in the volatility of the services sector, but not for the decline in the volatility of manufacturing.

### 4.4 The Role of Non-Homothetic Preferences

In this subsection I evaluate the importance of non-homotheticity in generating the results in table 6. To do this, I consider three alternative values of $\bar{s}$: the first 50% larger and the second 50% smaller than the value in table 5, with the third one set to zero (standard CES preferences). Figure 3 reports the pattern of the share of services generated with the benchmark calibration in table 5 and with the alternative values of $\bar{s}$. By increasing the value of $\bar{s}$ the model generates a larger amount of structural change.

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28 See the data appendix for a description of sectoral value added series.

29 The share of services in the model is computed as $p_s c_s / (p_s c_s + p_m c_m)$.

30 Note that even in the case in which $\bar{s} = 0$ there is structural change. This is generated by the mechanism described in Ngai and Pissarides (2007), which works through the interaction of an elasticity of substitution between manufacturing and services consumption smaller than one, and faster value added TFP growth in manufacturing.
the quantitative results of the model for the different values of $\bar{s}$. When $\bar{s}$ is large (0.0081), the share of services increases from 0.25 to 0.67. As the average share of services in the first part of the transition is smaller than in the baseline case, GDP growth and volatility are larger with respect to benchmark (3.14% versus 2.55% and 4.92% versus 3.82%). The share of services is smaller than in the baseline case also in the second part of the transition, so GDP growth and volatility are also larger with respect to benchmark (2.37% versus 2.16% and 3.73% versus 3.28%). In addition, the change in the structure of the economy between the first and the second part of the transition is more dramatic with respect to benchmark, so that the ratio of GDP growth and volatility between the first and the second part of the transition is larger than when $\bar{s} = 0.0054$ (1.32 versus 1.18 and 1.32 versus 1.16). The opposite reasoning applies to the case in which $\bar{s} = 0.0027$, that implies a share of services increasing from 0.59 to 0.70. The smaller increase in the share implies that the differences in GDP growth and volatility generated over the development path (a 9% larger growth and a 6% larger volatility in the first part of the transition) are also smaller than those in table 6. When $\bar{s} = 0$, the amount of structural change in the model is quantitatively negligible, and growth and volatility do not change along the growth path.

Consider now the effect of $\bar{s}$ on the volatility of individual sectors. First, note that when $\bar{s} = 0$ the manufacturing sector is the most volatile, reflecting the larger value added TFP volatility (due to the larger intensity of intermediates) with respect to services. By increasing $\bar{s}$, the volatility of manufacturing remains similar across simulations and along the growth path of each simulation. Instead, the volatility of services is systematically affected by the value of $\bar{s}$. First, when $\bar{s}$ increases, the volatility of services in the model also increases. Second, the larger $\bar{s}$, the larger the decline in the volatility of services along the growth path.

Results in table 7 suggest that, in the model with non-homothetic preferences, there are two effects at play in reducing volatility. One is the effect extensively described in subsection 4.4, that works through the endogenous reduction in the aggregate TFP multiplier due to intermediate goods. The other is the change in the income elasticities of consumption that occur as income grows. To see this, note that by using (7) and (8) it is possible to derive the income elasticity of manufacturing and services consumption. These are

$$\xi_{cm,t} = \frac{V_{m,t}}{V_{m,t} + \bar{s} (p_{s,t}/p_{m,t})},$$

28
Figure 3: Transition path of the share of services in GDP for different values of $\bar{s}$. Remaining parameter values are those reported in table 5.

and

$$
\xi_{cs,t} = \frac{V_{m,t}}{V_{m,t} - \bar{s} \left( p_{s,t} / p_{m,t} \right)^{1-r} \left( \frac{b}{1-b} \right)^{1-r}},
$$

where I used the equality $V_m = w_n / p_m$ and $V_m$ represent the measure of income. Equations (18) and (19) show that the income elasticity of services is larger than one and that of manufacturing smaller than one at any time $t$. Figure 4 displays the evolution of these elasticities along the growth path as implied by the parametrization in table 5. In period 1 the income elasticity of services is 1.44, while that of manufacturing is 0.62. As income grows they both tend to one, reaching 1.04 (services) and 0.90 (manufacturing). These elasticities drive structural change in the long-run. However, they also affect the way the economy responds to short-run income shocks. At low levels of output, services are highly sensitive to income shocks due to the high elasticity of substitution. The opposite holds for manufacturing. As income grows the response of services to income declines while that of manufacturing increases.

In the general equilibrium, the change in volatility along the growth path depends both on the changing structure of the economy and on the evolving elasticities of substitution. Thus, the non-homotheticity parameter $\bar{s}$ plays a double role here. First, it determines the extent of structural change in the long-run and so the reduction in the multiplier due to intermediate goods. Second, it directly affects business cycles in the short-run and their
evolution along the development path through income elasticities. By depending on a single parameter the two effects cannot be isolated. Put it differently, to have structural change in the model one has to face evolving income elasticities, while to have constant elasticities one has to admit no structural change.\(^{31}\) Results in table 6 show that in calibrated model the interaction of the two effect deliver a substantial reduction in the volatility of services but not in that of manufacturing. Intuitively, this is due to the fact that for services both the reduction in aggregate volatility due to structural change and the reduction in the income elasticity contribute to reduce the volatility of the sector. Instead, for manufacturing the reduction in aggregate volatility is offset by the increase in the income elasticity, leaving the volatility of the sector unchanged along the growth path.

\(^{31}\)In principle one can set \(s\) to zero and impose a very low elasticity of substitution between manufacturing and services to generate structural change as in Ngai and Pissarides (2007) and at the same time have constant income elasticities. However, with the amount of TFP growth measured in the data it is not possible to generate a sufficient amount of structural change even for an elasticity of substitution close to zero.
4.5 Sector Specific TFP processes

As discussed above, the calibration strategy of the previous subsection allows to study how much of the differences in growth and volatility between UMI and HI economies can be accounted for by structural change when there is a common sectoral TFP process in the two sectors. However, it is a theoretical possibility that the effect of structural change in shaping GDP growth and volatility be dampened or even cancelled when TFP processes are different in the two sectors. For instance, if gross output TFP grows faster and is more volatile in services than manufacturing, a larger share of services in GDP could imply a faster and more volatile GDP growth. To investigate whether this possibility is of empirical relevance for the results of the paper, this subsection studies the performance of the model when the TFP process of each sector is calibrated to the data. I present simulations in which TFP processes are calibrated to U.S. manufacturing and services data. The U.S. experienced a large process of structural transformation between manufacturing and services during the second part of the past century, thus providing a representative source to calibrate sectoral TFP processes in a model of structural change.

In contrast with the previous subsection, the calibration strategy adopted here does not impose any restriction on GDP growth and volatility in the first part of the transition.
Stochastic processes of TFP in the two sectors are estimated from the data. Given the estimated processes, the remaining parameters to be calibrated are $\bar{s}$, $b$ and $\varphi$. With respect to the strategy adopted in the previous subsection, this approach also allows to compare with the data the levels of growth and volatility generated by the model when fed with actual TFP processes.

I compute series for $B_{m,t}$ and $B_{s,t}$ for the U.S. using the model’s production functions (3) and (4) and data for gross output, labor services and intermediate goods for the 1960-2005 period from Jorgenson dataset. Average gross output TFP growth is 0.0079 in manufacturing and 0.0082 in services. Thus, in this case the structural transformation towards services implies an increase in the size of the sector with the largest growth rate of sectoral TFP.

<table>
<thead>
<tr>
<th>Table 8: Calibrated Parameters for the U.S. Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\nu_m$</td>
</tr>
<tr>
<td>$\xi_m$</td>
</tr>
<tr>
<td>$\nu_s$</td>
</tr>
<tr>
<td>$\xi_s$</td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
<tr>
<td>$b$</td>
</tr>
<tr>
<td>$\bar{s}$</td>
</tr>
<tr>
<td>$\varphi$</td>
</tr>
<tr>
<td>$B_{m,1}$</td>
</tr>
<tr>
<td>$B_{s,1}$</td>
</tr>
<tr>
<td>$\gamma_{B,m}$</td>
</tr>
<tr>
<td>$\gamma_{B,s}$</td>
</tr>
<tr>
<td>$\rho_m$</td>
</tr>
<tr>
<td>$\rho_s$</td>
</tr>
<tr>
<td>$\sigma_m$</td>
</tr>
<tr>
<td>$\sigma_s$</td>
</tr>
</tbody>
</table>

Gross output TFP in manufacturing follows

$$B_{m,t} = B_{m,t-1}(1 + \gamma_{B,m})e^{z_m,t},$$

with $z_{m,t} = \rho_m z_{m,t-1} + \epsilon_{m,t}$, $\epsilon_{m,t} \sim N(0, \sigma_m^2)$ and i.i.d. over time, and gross output TFP in services is

$$B_{s,t} = B_{s,t-1}(1 + \gamma_{B,s})e^{z_s,t},$$

\[32\] Measures of TFP at the sector level often suggest that manufacturing TFP grows faster than services. This is due to the fact that in such cases a value added measure of TFP is considered.
with \( z_{s,t} = \rho_s z_{s,t-1} + \epsilon_{s,t}, \epsilon_{s,t} \sim N(0, \sigma_s^2) \) and i.i.d. over time.

Equation (20) implies that the shock \( z_{m,t} \) is the difference of two components

\[
z_{m,t} = \log(B_{m,t}) - \log(B_{m,t-1}) - \log(1 + \gamma_{B,m}),
\]

the realized growth rate of \( B_m \) at \( t \) and the trend growth rate. I construct series of \( z_{m,t} \) using (21), the series \( B_{m,t} \) and the average growth rate \( \gamma_{B,m} \) measured in the data. I then use the series of \( z_{m,t} \) so constructed to estimate

\[
z_{m,t} = \rho_m z_{m,t-1} + \epsilon_{m,t}.
\]

The estimated \( \rho_m \) is 0.20 and the standard deviation of residuals is \( \sigma_m = 0.0104 \). The same procedure for services delivers \( \rho_s = 0.32 \) and \( \sigma_s = 0.0078 \). As in the calibration in table 1, the initial period TFP is normalized to one in both sectors \( B_{m,1} = 1, B_{s,1} = 1 \). With the new processes for \( B_m \) and \( B_s \), the parameters \( \bar{s}, b \) and \( \varphi \) are calibrated to match a share of services in GDP of 0.46 in the initial period, of 0.69 in the final period and an average growth of per-capita labor services of 0.43%. The calibrated parameters are \( \bar{s} = 0.0082, b = 0.0549 \) and \( \varphi = 5.2147 \). Parameter values are reported in table 8.

The first row of table 9 reports results for UMI, the second row results for HI, and the third row reports the ratios between the two economies. In terms of growth rates, the difference between the two economies, 20%, is close to that in table 6. Thus, also in this case structural transformation generates a difference in growth rates larger than that observed in the data when computing simple averages (1.12). Instead, when considering the difference in growth between UMI and HI implied by the panel estimation, the model accounts for half of it (20% versus 42%). Also, as discussed above, the calibration strategy adopted in this subsection does not impose that growth and volatility in the model be those of UMI economies in the first part of the transition. Regardless of this, growth rates have a similar magnitude in the data of UMI and in the model in the first part of the transition.

The ratio of the standard deviations of GDP growth rates is 24% in the model while it is 25% in the data. In this case, the different size of the services sector can account for the entire difference in volatility between UMI and HI income economies. This is due to the larger gross output TFP volatility in manufacturing with respect to services, which implies that the effect of structural change on volatility is larger than in table 6. Also, the model...
Table 9: Counterfactual with U.S. gross output TFP processes

<table>
<thead>
<tr>
<th></th>
<th>Average per-capita GDP</th>
<th>Average per-capita GDP</th>
<th>Value added volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Growth rate</td>
<td>Data (av.)</td>
<td>Data (est.)</td>
</tr>
<tr>
<td>Upper Middle Income</td>
<td>Model</td>
<td>2.61%</td>
<td>2.57%</td>
</tr>
<tr>
<td>(Model first 41 years)</td>
<td>Data</td>
<td>2.57%</td>
<td>2.57%</td>
</tr>
<tr>
<td>High Income</td>
<td>Model</td>
<td>2.17%</td>
<td>2.30%</td>
</tr>
<tr>
<td>(Model last 41 years)</td>
<td>Data</td>
<td>2.30%</td>
<td>1.81%</td>
</tr>
<tr>
<td>Ratio UMI/HI</td>
<td>1.20</td>
<td>1.12</td>
<td>1.42</td>
</tr>
</tbody>
</table>

Table 10: Counterfactual with U.S. gross output TFP processes and adjusted volatility levels

<table>
<thead>
<tr>
<th></th>
<th>Average per-capita GDP</th>
<th>Average per-capita GDP</th>
<th>Value added volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Growth rate</td>
<td>Data (av.)</td>
<td>Data (est.)</td>
</tr>
<tr>
<td>Upper Middle Income</td>
<td>Model</td>
<td>2.59%</td>
<td>2.57%</td>
</tr>
<tr>
<td>(Model first 41 years)</td>
<td>Data</td>
<td>2.57%</td>
<td>2.57%</td>
</tr>
<tr>
<td>High Income</td>
<td>Model</td>
<td>2.15%</td>
<td>2.30%</td>
</tr>
<tr>
<td>(Model last 41 years)</td>
<td>Data</td>
<td>2.30%</td>
<td>1.81%</td>
</tr>
<tr>
<td>Ratio UMI/HI</td>
<td>1.20</td>
<td>1.12</td>
<td>1.42</td>
</tr>
</tbody>
</table>

accounts for roughly 80% of the differences in volatility between UMI and HI implied by the regression in (15) (24% versus 30%).

Consider now the volatility of individual sectors. In table 6, the volatility of services in the first part of the transition is larger than that of manufacturing, something at odds with the data. This, as shown in table 7 and discussed in sub-section 5.4 is due to the non-homotheticity of preferences. Table 9 shows that when TFP processes are calibrated to US data this counterfactual result disappears, and the manufacturing sector is more volatile than services along the transition path. Also, the decline in the volatility of services is very close to that in the data, while the volatility of manufacturing in the model is constant, as in table 6.

Table 9 also suggests that the model delivers a low level of volatility compared to the data, both in GDP and in individual sectors. This result has to be attributed to the fact that the US economy is less volatile than the average UMI and the average HI economy. The standard deviation of annual per-capita GDP growth rates in the US in the period 1970-2010 is 2.18%, compared to 3.82% of UMI and 3.06% of HI. Thus, by calibrating TFP process to the US the model delivers a low level of volatility. Consider now the following exercise. By keeping relative gross output TFP in manufacturing and services as estimated for the US
(0.0104/0.0078 = 1.3333), I feed the model with a level of TFP volatility that allows it to deliver the same volatility of GDP of UMI economies in the first part of the transition. Thus, with respect to table 8, I set \( \sigma_m = 0.0185 \) and \( \sigma_s = \sigma_m / 1.33 = 0.0139 \). Results are reported in table 10. GDP volatility in the second part of the transition is now similar to that of HI economies. Also, the volatility of the services sector is very close to the data for both UMI and HI along the transition path. However, the model delivers a low level of manufacturing volatility when compared to the data (4.54% versus 6.96% and 4.53% versus 5.79%). Also, as in previous simulations, the manufacturing sector does not display a decline of volatility along the transition path.

The numerical results of this section point to an important role of the sectoral composition for the growth rate and the volatility of GDP. The analysis suggests that, other conditions equal, economies that differ in the size of the services sector relative to manufacturing display different patterns of GDP growth and volatility. Put it differently, the transmission of gross output TFP to aggregate TFP changes as an economy develops. Thus, the growth performance of GDP should be evaluated by taking into account the size of manufacturing and services in the economy. A similar argument holds for GDP volatility.

5 Conclusion

In this paper, I present a theory linking the structural composition of an economy to its growth and volatility. In particular, I show that, given the different intensity of intermediates in production in manufacturing and in services, structural change towards services reduces GDP growth and volatility. Using the model to quantify the importance of the sectoral composition I also find that the structure of the economy has a quantitatively important effect in explaining GDP growth and volatility differences between High and Upper-Middle income economies. Thus, this paper represents an attempt to reconcile both cross-country and times series evidence on GDP growth and volatility in a unique environment.
Data Appendix

KLEMS dataset, 2008. This dataset provides harmonized data for 30 countries. These are the countries reported in figure 1 plus Malta, Luxemburg and the U.S. The first two are excluded for their size. For the U.S. I use data from Jorgenson Dataset, 2007, that reports a longer time period. The share of intermediate goods in gross output in manufacturing is computed as the total value of intermediate goods used in manufacturing divided the total value of gross output produced in manufacturing. The share of intermediate goods in gross output in services is accordingly constructed. The sectors used in manufacturing computations are 1) Agriculture, hunting and forestry, 2) Fishing, mining and quarrying, 3) Total Manufacturing, 4) Electricity, gas and water supply, 5) Construction. The sectors used in services computations are 6) Wholesale and retail trade, 7) Hotels and restaurants, 8) Transport, storage and communication, 9) Financial intermediation, 10) Real estate, renting and business activities, 11) Public administration and defense, 12) Education, 13) Health and social work, 14) Other community, social and personal services, 15) Private households with employed persons, 16), Extra-territorial organizations and bodies.

Communications, 30) Electric utilities (services), 31) Gas utilities (services), 32) Wholesale
and retail trade, 33) Finance, insurance and real estate, 34) Personal and business services,
35) Government enterprises.

**World Bank Data** *(World Development Indicators)*. The World Bank reports
time series for per-capita GDP growth, per-capita GDP level and the share of services in
GDP for a large set of countries and classifies these into High, Upper-Middle, Lower-Middle
and Low income categories. Using these data I construct an unbalanced panel for each
income group. These are the data used for estimations in tables 1 and 4. Using all countries
for which the growth rate of GDP is available from 1970 to 2012 I compute the standard
deviation of GDP growth over the period and regress it on the share of services in 1970,
1980, 1990, 2000 and 2010. These are the results reported in table 2. For table 3 I use the
same methodology but using only Upper-Middle and High income countries.

To construct figure 2 I use all Upper Middle and High income countries for which both the
share of services and the growth rate of per-capita GDP are reported from 1970 to 2010 in the
World Development Indicators. These are 15 Upper Middle Income: Algeria, Brazil, China,
Colombia, Dominican Republic, Ecuador, Fiji, Hungary, Malaysia, Mexico, South Africa,
Thailand, Tunisia, Turkey, Venezuela; and 19 High Income economies: Austria, Barbados,
Belgium, Chile, Denmark, Finland, Italy, Japan, South Korea, Luxembourg, Netherlands,
Norway, Portugal, Puerto Rico, Saudi Arabia, Spain, Sweden, U.K. and U.S. For each year
I take the average share of services across countries in each group and report the two time
series so obtained in figure 2. Also, the statistics on per-capita GDP growth reported in
tables 6, 7, 9 and 10 are computed by taking averages across countries in each year and then
the average across time of per-capita GDP growth rates of individual countries. Statistics
on per-capita GDP volatility reported in tables 6, 7, 9 and 10 are computed by taking the
standard deviation of per-capita GDP growth for each country over the 1970-2010 period
and then averaging across countries. The balanced panel of 15 Upper Middle Income and
19 High Income economies is used to run the panel regression (1) in subsection 4.1. The
estimated coefficient is used to compute the difference in per-capita GDP growth between
Upper Middle and High income economies reported in the third column of tables 6, 7, 9 and
10. To estimate (15) I use the vector of standard deviations of per-capita GDP of the 34

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33 Considering time spans that start before 1970 and/or end after 2010 reduces the number of available countries.
countries over the 1970-2010 period and regress it on the vector of average shares of services
during the same period. Finally, using real value added data from the World Development
Indicators I compute the volatilities of manufacturing and services reported in tables 6, 7,
9 and 10. Data are available since 1971. The methodology and the countries used are the
same as those used in computing statistics for per-capita GDP. For some countries there
are missing values in a subset of years. Where data are available I use KLEMS time series
for real value added in manufacturing and services. When data are missing the standard
deviation is computed for a time span shorter than 1971-2010.34

Appendix A: Relative Price and Aggregate Production Functions

The maximization problem of the representative firm in manufacturing is

$$\max_{N_m, M_m, S_m} [p_m G_m - w N_m - p_m M_m - p_s S_m]$$

subject to $G_m = B_m N_m^{1-\varepsilon_m} (M_m^{1-\varepsilon_m} S_m^{1-\varepsilon_m})^{1-\nu_m}$,

where $w$ is the wage rate, $p_m$ the price of the manufacturing good and $p_s$ the price of services.
The problem of the firm in services is

$$\max_{N_s, M_s, S_s} [p_s G_s - w N_s - p_m M_s - p_s S_s]$$

subject to $G_s = B_s N_s^{1-\varepsilon_s} (M_s^{1-\varepsilon_s} S_s^{1-\varepsilon_s})^{1-\nu_s}$.

As the gross output production function in the two sectors is Cobb-Douglas, it is possible
to derive a net production function, defined as the amount of gross output produced in one
sector minus the amount of inputs produced and used in the same sector. The net production
function in the manufacturing sector is obtained by solving

$$Y_m = \max_{M_m} \left\{ B_m N_m^{1-\varepsilon_m} (M_m^{1-\varepsilon_m} S_m^{1-\varepsilon_m})^{1-\nu_m} - M_m \right\},$$

and it is equal to
\[ Y_m = \Phi_{m1} B_m \left[ I_{\nu_m(1-\nu_m)} \right] N_m \left[ I_{\nu_m(1-\nu_m)} \right] S_m \left[ I_{\nu_m(1-\nu_m)} \right], \]
where \( \Phi_{m1} = \left[ 1 - \varepsilon_m (1 - \nu_m) \right] \left[ \varepsilon_m (1 - \nu_m) \right] \left[ I_{\nu_m(1-\nu_m)} \right]. \) Equation (24) can be re-written as
\[ Y_m = A_m N_m \theta \theta^{1-\theta}, \] (25)
where
\[ A_m = \Phi_{m1} B_m \left[ I_{\nu_m(1-\nu_m)} \right], \] (26)

0 < \theta < 1 is equal to \( \frac{\nu_m}{1-\varepsilon_m(1-\nu_m)} \) and \( S_m = S. \) The problem of the firm in the manufacturing sector becomes
\[ \max_{N_m,S} \left[ p_m Y_m - w N_m - p_s S \right] \] (27)
subject to (25) and (26).

By using the same derivation, the net production function in the services sector is given by
\[ Y_s = A_s N_s \gamma M^{1-\gamma}, \] (28)
where 0 < \gamma < 1 is equal to \( \frac{\nu_s}{1-\varepsilon_s(1-\nu_s)} \), \( N_s \) is the amount of labor and \( M \) is the amount of manufacturing used as intermediate good in the services sector, with
\[ A_s = \Phi_{s1} B_s \left[ I_{\nu_s(1-\nu_s)} \right], \] (29)
and \( \Phi_{s1} = \left[ 1 - \varepsilon_s (1 - \nu_s) \right] \left[ \varepsilon_s (1 - \nu_s) \right] \left[ I_{\nu_s(1-\nu_s)} \right]. \)

The problem of the representative firm in services becomes
\[ \max_{N_s,M} \left[ p_s Y_s - w N_s - p_m M \right] \] (30)
subject to (28) and (29).

The production functions (25) and (28) and competitive markets imply that in equilibrium prices need to be
\[ p_m = \frac{w^\theta p_s^{1-\theta}}{\Phi_{m2} A_m}, \] (31)
and
\[ p_s = \frac{w^\gamma p_m^{1-\gamma}}{\Phi_{s2} A_s}, \] (32)
where $\Phi_{m_2} = \theta^\theta (1 - \theta)^{1 - \theta}$ and $\Phi_{s_2} = \gamma^\gamma (1 - \gamma)^{1 - \gamma}$. The relative price of the two goods is

$$
\frac{p_s}{p_m} = \frac{(\Phi_{m_2} A_m)^{\gamma + \theta - \gamma_1}}{(\Phi_{s_2} A_s)^{\gamma + \theta - \gamma_1}}. 
$$

(33)

By substituting (26) and (29), (33) can be rewritten as

$$
\frac{p_s}{p_m} = \left( \frac{\Phi_{m_1} \Phi_{m_2} B_m^{\frac{1}{1 - \nu_m (1 - \nu_m)}}}{\Phi_{s_1} \Phi_{s_2} B_s^{\frac{1}{1 - \nu_s (1 - \nu_s)}}} \right)^{\gamma + \theta - \gamma_1},
$$

Finally, using $\theta = \frac{\nu_m}{1 - \nu_m (1 - \nu_m)}$ and $\gamma = \frac{\nu_s}{1 - \nu_s (1 - \nu_s)}$,

$$
\frac{p_s}{p_m} = \Omega(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) \left( \frac{B_m^{\nu_m}}{B_s^{\nu_s}} \right)^{\frac{\nu_m [1 - \varepsilon_s (1 - \nu_s)] + \nu_s [1 - \varepsilon_m (1 - \nu_m)] - \nu_s \nu_m}{\nu_m [1 - \varepsilon_s (1 - \nu_s)] + \nu_s [1 - \varepsilon_m (1 - \nu_m)] - \nu_s \nu_m},
$$

(34)

where

$$
\Omega(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) = \frac{(\Phi_{m_1} \Phi_{m_2})^{\frac{\nu_m [1 - \varepsilon_s (1 - \nu_s)] + \nu_s [1 - \varepsilon_m (1 - \nu_m)] - \nu_s \nu_m}{\nu_m [1 - \varepsilon_s (1 - \nu_s)] + \nu_s [1 - \varepsilon_m (1 - \nu_m)] - \nu_s \nu_m}}{(\Phi_{s_1} \Phi_{s_2})^{\frac{\nu_m [1 - \varepsilon_s (1 - \nu_s)] + \nu_s [1 - \varepsilon_m (1 - \nu_m)] - \nu_s \nu_m}{\nu_m [1 - \varepsilon_s (1 - \nu_s)] + \nu_s [1 - \varepsilon_m (1 - \nu_m)] - \nu_s \nu_m}}.
$$

To find the aggregate production function in manufacturing units at time $t$ it is useful to rely again on the net production functions, and solve

$$
\max_{N_m, M, S} \left[ A_m N_m^{\theta} S^{1 - \theta} - M \right]
$$

subject to

$$
A_s (n - N_m) \gamma M^{1 - \gamma} = S,
$$

where $n$ is the total amount of labor services used in production in the economy. Note that (35) corresponds to a reduced form of problem (9) in the main text (with $c_s = 0$), in which the first order conditions with respect to $M_m$ and $S_s$ already hold. The solution to problem (35) determines the maximum amount of manufacturing that can be consumed in the economy when the services sector produces only the intermediate goods needed in the manufacturing sector, $S_t$, that is, when services is only an intermediate goods sector. This is given by

$$
V_m = \Phi_{m_3} A_m^{\frac{1}{\gamma + \theta - \gamma_1}} A_s^{\frac{1}{\gamma + \theta - \gamma_1}} n,
$$

(36)

with $\Phi_{m_3} = [1 - (1 - \theta)(1 - \gamma)][(1 - \theta)(1 - \gamma)]^{\frac{1 - \theta + \gamma + \gamma_1 \gamma}{\gamma + \theta - \gamma_1 \gamma}} \left( \frac{\theta}{\gamma + \theta - \gamma_1 \gamma} \right)^{\frac{\gamma (1 - \theta)}{\gamma + \theta - \gamma_1 \gamma}}$. By substituting the definitions of $A_m$ and $A_s$, (36) becomes

$$
V_m = \Phi_{m_3} \Phi_{m_1}^{\frac{1}{\gamma + \theta - \gamma_1}} \Phi_{s_1}^{\frac{1}{\gamma + \theta - \gamma_1}} B_m^{\frac{1}{1 - \nu_m (1 - \nu_m)}} B_s^{\frac{1}{1 - \nu_s (1 - \nu_s)}} n,
$$
and by defining $\Theta_m = \Phi_{\frac{1}{m_1}}^{\gamma_1 \theta_1} \Phi_m \Phi_{\frac{1}{m_2}}^{\gamma_2 \theta_2}$ and using the definitions of $\theta$ and $\gamma$ it is possible to write (36) as

$$V_m = \Theta_m(\nu_m, \nu_s, \epsilon_m, \epsilon_s)B_m f_1(\nu_m, \nu_s, \epsilon_m, \epsilon_s)B_s f_2(\nu_m, \nu_s, \epsilon_m, \epsilon_s)n,$$

which is equation (10) in the main text. In (37)

$$f_1 = \frac{1 - \epsilon_s(1 - \nu_s)}{\nu_m[1 - \epsilon_s(1 - \nu_s)] + \nu_s[1 - \epsilon_m(1 - \nu_m)] - \nu_m \nu_s},$$

and

$$f_2 = \frac{(1 - \epsilon_m)(1 - \nu_m)}{\nu_m[1 - \epsilon_s(1 - \nu_s)] + \nu_s[1 - \epsilon_m(1 - \nu_m)] - \nu_m \nu_s}.$$

By dividing (37) by (34) it is possible to obtain

$$V_s = \Theta_s(\nu_m, \nu_s, \epsilon_m, \epsilon_s)B_m f_3(\nu_m, \nu_s, \epsilon_m, \epsilon_s)B_s f_4(\nu_m, \nu_s, \epsilon_m, \epsilon_s)n,$$

which is equation (11) in the paper. In (38) $\Theta_s = \Theta_m/\Omega$,

$$f_3 = \frac{(1 - \epsilon_s)(1 - \nu_s)}{\nu_m[1 - \epsilon_s(1 - \nu_s)] + \nu_s[1 - \epsilon_m(1 - \nu_m)] - \nu_m \nu_s},$$

and

$$f_4 = \frac{1 - \epsilon_m(1 - \nu_m)}{\nu_m[1 - \epsilon_s(1 - \nu_s)] + \nu_s[1 - \epsilon_m(1 - \nu_m)] - \nu_m \nu_s}.$$

### Appendix B: Omitted Proofs

**Proof of Proposition 1.** Taking logs of (12) and (13) and the difference between two consecutive periods, the growth rate of aggregate TFP in the two cases is $\gamma_{tp,m} = f_1 \gamma_{B_m} + f_2 \gamma_{B_s}$ and $\gamma_{tp,s} = f_3 \gamma_{B_m} + f_4 \gamma_{B_s}$, respectively, where $\gamma_{B_m} = \frac{B_{m,t}}{B_{m,t-1}} - 1$ and $\gamma_{B_s} = \frac{B_{s,t}}{B_{s,t-1}} - 1$ and the approximation $\log(1 + x) \approx x$ has been used. When the variance of $z_t$ is zero, $z_t = 0$ at each $t$ and the assumption of a common growth factor of sectoral TFP in the two sectors implies $\gamma_{B_m} = \gamma_{B_s} = \gamma_B$. Thus, the growth rate of aggregate TFP is

$$\gamma_{tp,m} = (f_1 + f_2) \gamma_B,$$

in (12) and

$$\gamma_{tp,s} = (f_3 + f_4) \gamma_B.$$
in (13). It follows that, because $\gamma_B > 0$, if $f_1 + f_2 > f_3 + f_4$ then $\gamma_{tfp,m} > \gamma_{tfp,s}$. To prove that $f_1 + f_2 > f_3 + f_4$ when $1 - \nu_m > 1 - \nu_s$, note that the common denominator of $f_1, f_2, f_3$ and $f_4$ is always positive. This is evident by re-writing it as $\nu_m (1 - \nu_s) + \nu_s (1 - \epsilon_m) + \nu_m \nu_s \epsilon_s + \nu_m \nu_s \epsilon_m$. Thus, by using the numerators, $f_1 + f_2 > f_3 + f_4$ reads

$$1 = \nu_s (1 - \nu_s) + (1 - \epsilon_m) (1 - \nu_m) > (1 - \nu_s) (1 - \nu_m) + 1 - \epsilon_m (1 - \nu_m)$$

which can be simplified to obtain $1 - \nu_m > 1 - \nu_s$.

**Proof of Proposition 2.** Taking logs of (14) and using $\log(1 + x) \simeq x$, $\gamma_{B_m} = \gamma_{B_s} = \gamma_B + z_t$. The standard deviation (sd) of growth rates $\gamma_{tfp,m} = f_1 \gamma_{B_m} + f_2 \gamma_{B_s}$ and $\gamma_{tfp,s} = f_3 \gamma_{B_m} + f_4 \gamma_{B_s}$ then reads

$$sd(\gamma_{tfp,m}) = (f_1 + f_2) \, sd(\gamma_B + z_t) = (f_1 + f_2) \, sd(z_t),$$

and

$$sd(\gamma_{tfp,s}) = (f_3 + f_4) \, sd(\gamma_B + z_t) = (f_3 + f_4) \, sd(z_t).$$

Thus $sd(\gamma_{tfp,m}) > sd(\gamma_{tfp,s})$ when $f_1 + f_2 > f_3 + f_4$, which holds as $1 - \nu_m > 1 - \nu_s$.

**Proof of Proposition 3.** As the consumption index is given by $c = c_m^b c_s^{1-b}$, the amount of labor in equilibrium is

$$n = \frac{1}{1 + \varphi}. \tag{41}$$

Also, as output coincides with consumption, $y = c$, its growth rate is given by

$$\gamma_y = b \gamma_{cm} + (1 - b) \gamma_{cs}$$

where $\gamma_{cm}$ and $\gamma_{cs}$ are the growth rates of manufacturing and services consumption and again the approximation $\log(1 + x) \simeq x$ has been used. In equilibrium $\frac{c_m}{V_m} = b$ and $\frac{c_s}{V_s} = 1 - b$, so the growth rate of $c_m$ is given by the growth rate of $V_m$, which is solely determined by $\gamma_{tfp,m}$ as labor is constant from (41). Equivalently, the growth rate of $c_s$ is given by the growth rate of $V_s$, which coincides with $\gamma_{tfp,s}$. It follows that the growth rate of output can be written as

$$\gamma_y = b \gamma_{tfp,m} + (1 - b) \gamma_{tfp,s}. \tag{42}$$

As $\gamma_{tfp,m} > \gamma_{tfp,s}$, because $1 - \nu_m > 1 - \nu_s$, the larger is $b$, the larger is the growth rate of the economy.
Proof of Proposition 4. Taking logs of (14) and using \( \log(1+x) \approx x \), \( \gamma_{B_m} = \gamma_{B_s} = \gamma_B + z_t \). Thus, \( \gamma_{tfp,m} = (f_1 + f_2) (\gamma_B + z_t) \) and \( \gamma_{tfp,s} = (f_3 + f_4) (\gamma_B + z_t) \) and (42) can be written as \( \gamma_y = b (f_1 + f_2) (\gamma_B + z_t) + (1 - b) (f_3 + f_4) (\gamma_B + z_t) = [b (f_1 + f_2) + (1 - b) (f_3 + f_4)] (\gamma_B + z_t) \). Taking the standard deviation of the last expression

\[
sd(\gamma_y) = [b (f_1 + f_2) + (1 - b) (f_3 + f_4)] sd(\gamma_B + z_t) = [b (f_1 + f_2) + (1 - b) (f_3 + f_4)] sd(z_t).
\]

As \( f_1 + f_2 > f_3 + f_4 \), because \( 1 - \nu_m > 1 - \nu_s \), the volatility of output increases with the size of the manufacturing sector \( b \).

**Appendix C: Real GDP**

To construct the model’s counterpart of real GDP in the data it is first needed to obtain real value added in the two sectors, \( y_{m,t} \) and \( y_{s,t} \). This is given by

\[
y_{m,t} = \frac{p_{m,t} c_{m,t}}{p_{ym,t}} \quad \text{and} \quad y_{s,t} = \frac{p_{s,t} c_{s,t}}{p_{ys,t}}.
\]

Here \( p_{ym,t} \) and \( p_{ys,t} \) are the value added deflators in manufacturing and services. To derive \( p_{ym,t} \), consider again the maximization problem of the manufacturing firm

\[
\max_{N_m, M_m, S_m} [p_m G_m - w N_m - p_m M_m - p_s S_m] \quad \text{(43)}
\]

subject to \( G_m = B_m N_m^{\nu_m} (M_m^{\varepsilon_m} S_m^{1 - \varepsilon_m})^{1 - \nu_m} \).

Define

\[
R_m = M_m^{\varepsilon_m} S_m^{1 - \varepsilon_m}, \quad \text{(44)}
\]

as the intermediate goods index in the manufacturing sector. Given the Cobb-Douglas form of this index, with competitive markets the price of \( R_m \) is

\[
p_r = \frac{p_m^{\varepsilon_m} p_s^{1 - \varepsilon_m}}{\varepsilon_m (1 - \varepsilon_m)^{1 - \varepsilon_m}}. \quad \text{(45)}
\]

Thus, problem (43) can be written as

\[
\max_{N_m, R_m} [p_m G_m - w N_m - p_r R_m] \quad \text{(46)}
\]

subject to \( G_m = B_m N_m^{\nu_m} R_m^{1 - \nu_m} \).

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The first order condition of (46) with respect to $R_m$ delivers the following condition

$$R_m = (1 - \nu_m)^{1/r_m} \left( \frac{p_m}{p_r} \right)^{1/r_m} B_m^{v_m} N_m.$$  

(47)

By plugging (47) into (46) it is possible to obtain the reduced form problem

$$\max_{K_m, N_m} \left[ p_{vm} VA_m - w N_m \right]$$

subject to

$$p_{vm} VA_m = \nu_m (1 - \nu_m)^{1/v_m} \left( \frac{p_m}{p_r^{1-\nu_m}} \right)^{1/v_m} B_m^{v_m} N_m.$$  

Here $p_{vm} VA_m$ represents nominal value added. Real value added $VA_m$ is defined, as in Sato (1976), as the contribution to gross output growth of primary inputs (here only labor) and technical change. It follows that the real value added function is given by $VA_m = B_m^{v_m} N_m$ and its price is $p_{vm} = \nu_m (1 - \nu_m)^{1/v_m} \left( \frac{p_m}{p_r^{1-\nu_m}} \right)^{1/v_m}$. By using (45) in the last expression, the price of manufacturing value added is obtained. The value added price for services is accordingly constructed. Aggregate real value added at $t$, which is the model’s counterpart of real GDP in the data, is computed as a chain-weighted Fisher index of sectoral value added. This is the same concept suggested by NIPA and used to construct the U.S. real GDP series.
References


